# Worksheet 12.2
# Habitable Zones Near and Far

1. This problem is not really related to Habitable zones, but it is related to stellar scaling relations, which you will use on the following questions.

   (a) Stars are primarily made of hydrogen. Stars fuse hydrogen to produce the energy needed to hold them up against gravity. How does the fuel supply of a star scale with its mass (don't over-think this)

   (b) How quickly does a star use this fuel, and how does this rate scale with mass?

   (c) How does the lifetime, $\tau$, of a star scale with mass?

2. The Earth resides in a "Goldilocks Zone" or habitable zone (HZ) around the Sun. At our semimajor axis we receive just enough Sunlight to prevent the planet from freezing over and not too much to boil off our oceans. Not too cold, not too hot. Just right. In this problem we'll calculate how the temperature of a planet, $T_p$, depends on the properties of the central star and the orbital properties of the planet.

   (a) Draw the Sun on the left, and a planet on the right, separated by a distance $a$. The planet has a radius $R_p$ and temperature $T_p$. The star has a radius $R_\star$ and a luminosity $L_\star$ and a temperature $T_{\rm eff}$.

   (b) Due to energy conservation, the amount of energy received per unit time by the planet is equal to the energy emitted isotropically under the assumption that it is a blackbody.

      How much energy per time does the planet receive from the star?

      How much energy per time does the Earth radiate as a blackbody?

   (c) Set these two quantities equal to each other and solve for $T_P$.

   (d) How does the temperature change if the planet were much larger or much smaller?

   (e) Not all of the energy incident on the planet will be absorbed. Some fraction, $A$, will be reflected back out into space. How does this affect the amount of energy received per unit time, and thus how does this affect $T_p$?

3. In this problem we'll figure out how the habitable zone distance, $a_{\rm HZ}$, depends on stellar mass. Recall the average mass-luminosity relation we derived earlier, as well as the mass-radius relation for stars on the main sequence. If you don't recall, this is a good time to practice something that will very likely show up on the final!

   (a) Express $a_{\rm HZ}$ in terms of stellar properties as a scaling relationship, using squiggles instead of equal signs and ditching constants.

   (b) Replace the stellar parameters with their dependence on stellar mass, such that $a_{\rm HZ} \sim M_\star^\alpha$. Find $\alpha$.

   (c) If the Sun were half as massive and the Earth had the same equilibrium temperature, how many days would our year contain?



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